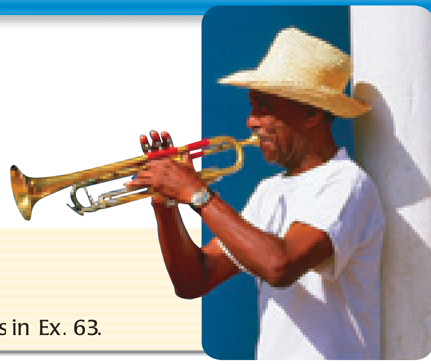
7.5 Apply Properties of Logarithms



Before You evaluated logarithms.

Now You will rewrite logarithmic expressions.

Why? So you can model the loudness of sounds, as in Ex. 63.

Key Vocabulary
- **base,** *p. 10*

KEY CONCEPT *For Your Notebook*

Properties of Logarithms

Let b, m, and n be positive numbers such that $b \neq 1$.

Product Property $\log_b mn = \log_b m + \log_b n$

Quotient Property $\log_b \frac{m}{n} = \log_b m - \log_b n$

Power Property $\log_b m^n = n \log_b m$

EXAMPLE 1 Use properties of logarithms

Use $\log_4 3 \approx 0.792$ and $\log_4 7 \approx 1.404$ to evaluate the logarithm.

AVOID ERRORS
Note that in general $\log_b \frac{m}{n} \neq \frac{\log_b m}{\log_b n}$ and $\log_b mn \neq (\log_b m)(\log_b n)$.

a. $\log_4 \frac{3}{7} = \log_4 3 - \log_4 7$ **Quotient property**

$\approx 0.792 - 1.404$ **Use the given values of $\log_4 3$ and $\log_4 7$.**

$= -0.612$ **Simplify.**

b. $\log_4 21 = \log_4 (3 \cdot 7)$ **Write 21 as $3 \cdot 7$.**

$= \log_4 3 + \log_4 7$ **Product property**

$\approx 0.792 + 1.404$ **Use the given values of $\log_4 3$ and $\log_4 7$.**

$= 2.196$ **Simplify.**

c. $\log_4 49 = \log_4 7^2$ **Write 49 as 7^2.**

$= 2 \log_4 7$ **Power property**

$\approx 2(1.404)$ **Use the given value of $\log_4 7$.**

$= 2.808$ **Simplify.**

✓ **GUIDED PRACTICE** for Example 1

Use $\log_6 5 \approx 0.898$ and $\log_6 8 \approx 1.161$ to evaluate the logarithm.

1. $\log_6 \frac{5}{8}$ **−0.263** **2.** $\log_6 40$ **2.059** **3.** $\log_6 64$ **2.322** **4.** $\log_6 125$ **2.694**

1 PLAN AND PREPARE

Warm-Up Exercises

Transparency Available

Evaluate the logarithm.

1. $\log_5 625$ **4**

2. $\log 0.00001$ **−5**

3. $\log_{32} 2$ $\frac{1}{5}$

4. $\log_{36} \frac{1}{6}$ $-\frac{1}{2}$

5. $\log_8 4$ $\frac{2}{3}$

Notetaking Guide

Transparency Available

Promotes interactive learning and notetaking skills, pp. 199–201.

Pacing

Basic: 1 day
Average: 1 day
Advanced: 1 day
Block: 0.5 block with 7.4
- See Teaching Guide/Lesson Plan.

2 FOCUS AND MOTIVATE

Essential Question

Big Idea 2, p. 477

How can you use a calculator to evaluate a logarithm when the base is not 10 or e? **Tell students they will learn how to answer this question by learning to use the change-of-base formula.**

NCTM STANDARDS

Standard 2: Analyze situations using algebraic symbols

Standard 6: Solve problems in math and other contexts

Resource Planning Guide

Chapter Resource Book
- Teaching Guide/Lesson Plan (pp. 51–52)
- Practice levels A, B, C (pp. 54–56)
- Study Guide (pp. 57–58)
- Catch-up for Absent Students (p. 59)
- Application (p. 60)
- Challenge (p. 61)

Workbooks
- Notetaking Guide (pp. 199–201)
- Practice Workbook (pp. 116–117)

Teaching Options
- **Power Presentations CD-ROM** provides dynamic electronic teaching resources for the classroom.
- **Activity Generator CD-ROM** provides editable activities for all ability levels.

Interactive Technology
- Easy Planner
- Power Presentations CD-ROM
- Activity Generator CD-ROM
- Animated Algebra
- Test Generator CD-ROM
- Online Quiz
- eWorkbook
- eEdition
- @HomeTutor

Resources for English Learners
- Quick Reference for English Learners
- Spanish Study Guide
- Multi-Language Visual Glossary
- Student Resources in Spanish

See also the Algebra 2 Toolkit for more strategies for meeting individual needs.

Motivating the Lesson

You can use properties of logarithms to decide how many times as great the noise level is at a rock concert as the noise level at a string quartet concert.

3 TEACH

Extra Example 1

Use $\log_3 12 \approx 2.262$ and $\log_3 2 \approx 0.631$ to evaluate the logarithm.

a. $\log_3 6$ **1.631**

b. $\log_3 24$ **2.893**

c. $\log_3 32$ **3.155**

Extra Example 2

Expand $\log_7 \frac{3x^2}{5y^3}$.

$\log_7 3 + 2\log_7 x - \log_7 5 - 3\log_7 y$

Extra Example 3

Which of the following is equivalent to $\ln 8 + 2\ln 5 - \ln 10$? **D**

(A) ln 4

(B) ln 8

(C) ln 18

(D) ln 20

Key Question to Ask for Examples 2 and 3

- What does it mean to "expand" or "condense" a logarithmic expression? **Expanding a logarithmic expression means to break down a single logarithm into the sum and/or difference of two or more logarithms with the same base as the original expression. Condensing a logarithmic expression is the reverse of expanding, so the result is a single logarithm.**

REWRITING EXPRESSIONS You can use the properties of logarithms to expand and condense logarithmic expressions.

REWRITE EXPRESSIONS When you are expanding or condensing an expression involving logarithms, you may assume any variables are positive.

EXAMPLE 2 Expand a logarithmic expression

Expand $\log_6 \frac{5x^3}{y}$.

$\log_6 \frac{5x^3}{y} = \log_6 5x^3 - \log_6 y$ **Quotient property**

$= \log_6 5 + \log_6 x^3 - \log_6 y$ **Product property**

$= \log_6 5 + 3\log_6 x - \log_6 y$ **Power property**

EXAMPLE 3 Standardized Test Practice

Which of the following is equivalent to log 9 + 3 log 2 − log 3?

(A) log 8 (B) log 14 (C) log 18 (D) log 24

Solution

$\log 9 + 3\log 2 - \log 3 = \log 9 + \log 2^3 - \log 3$ **Power property**

$= \log (9 \cdot 2^3) - \log 3$ **Product property**

$= \log \frac{9 \cdot 2^3}{3}$ **Quotient property**

$= \log 24$ **Simplify.**

▶ The correct answer is D. (A) (B) (C) (D)

GUIDED PRACTICE for Examples 2 and 3

5. Expand $\log 3x^4$. **$\log 3 + 4\log x$**

6. Condense $\ln 4 + 3\ln 3 - \ln 12$. **ln 9**

CHANGE-OF-BASE FORMULA Logarithms with any base other than 10 or e can be written in terms of common or natural logarithms using the *change-of-base formula*. This allows you to evaluate any logarithm using a calculator.

KEY CONCEPT *For Your Notebook*

Change-of-Base Formula

If a, b, and c are positive numbers with $b \neq 1$ and $c \neq 1$, then:

$$\log_c a = \frac{\log_b a}{\log_b c}$$

In particular, $\log_c a = \frac{\log a}{\log c}$ and $\log_c a = \frac{\ln a}{\ln c}$.

Differentiated Instruction

Inclusion Instead of having to apply the properties of logarithms shown on page 507 strictly from memory, highlight the division bar and the subtraction sign in the quotient property. Ask students to notice that this short horizontal line is used twice here, so it will help them keep the operations used separate from those in the product property.

See also the Algebra 2 Toolkit for more strategies.

EXAMPLE 4 Use the change-of-base formula

Evaluate $\log_3 8$ using common logarithms and natural logarithms.

Solution

Using common logarithms: $\log_3 8 = \frac{\log 8}{\log 3} \approx \frac{0.9031}{0.4771} \approx 1.893$

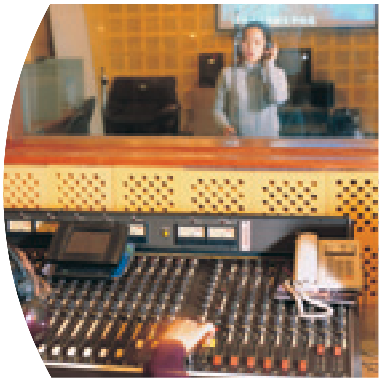
Using natural logarithms: $\log_3 8 = \frac{\ln 8}{\ln 3} \approx \frac{2.0794}{1.0986} \approx 1.893$

EXAMPLE 5 Use properties of logarithms in real life

SOUND INTENSITY For a sound with intensity I (in watts per square meter), the loudness $L(I)$ of the sound (in decibels) is given by the function

$$L(I) = 10 \log \frac{I}{I_0}$$

where I_0 is the intensity of a barely audible sound (about 10^{-12} watts per square meter). An artist in a recording studio turns up the volume of a track so that the sound's intensity doubles. By how many decibels does the loudness increase?



Solution

Let I be the original intensity, so that $2I$ is the doubled intensity.

Increase in loudness	$= L(2I) - L(I)$	**Write an expression.**
	$= 10 \log \frac{2I}{I_0} - 10 \log \frac{I}{I_0}$	**Substitute.**
	$= 10\left(\log \frac{2I}{I_0} - \log \frac{I}{I_0}\right)$	**Distributive property**
	$= 10\left(\log 2 + \log \frac{I}{I_0} - \log \frac{I}{I_0}\right)$	**Product property**
	$= 10 \log 2$	**Simplify.**
	≈ 3.01	**Use a calculator.**

▶ The loudness increases by about 3 decibels.

✓ GUIDED PRACTICE for Examples 4 and 5

Use the change-of-base formula to evaluate the logarithm.

7. $\log_5 8$ **about 1.292**
8. $\log_8 14$ **about 1.269**
9. $\log_{26} 9$ **about 0.674**
10. $\log_{12} 30$ **about 1.369**
11. **WHAT IF?** In Example 5, suppose the artist turns up the volume so that the sound's intensity triples. By how many decibels does the loudness increase? **about 4.771 decibels**

Extra Example 4

Evaluate $\log_6 24$ using common logarithms and natural logarithms. **approximately 1.774; approximately 1.774**

Teaching Strategy

Before discussing Example 4, give students specific examples of using the change-of-base formula to rewrite logarithms in bases other than 10 or e as quotients of common or natural logarithms.

Extra Example 5

The Richter scale is used to measure the magnitude of earthquakes. If an earthquake has intensity I, then its magnitude on the Richter scale, R, is given by the function $R(I) = \log \frac{I}{I_0}$, where I_0 is the intensity of a barely felt earthquake. If the intensity of one earthquake is 50 times that of another, how many points greater is the bigger earthquake on the Richter scale? **about 1.7 points**

Closing the Lesson

Have students summarize the major points of the lesson and answer the Essential Question: How can you use a calculator to evaluate a logarithm when the base is not 10 or e?

- **The product, quotient, and power properties of logarithms are directly related to the corresponding properties of exponents.**
- **The properties of logarithms can be used to expand or condense logarithmic expressions.**

The change-of-base formula allows you to evaluate a logarithm with any base by finding the quotient of two common logarithms or of two natural logarithms. Thus, you can rewrite the logarithm using the change-of-base formula and then evaluate the resulting expression using a calculator.